Example 2.2 [Ribbed slab design]

A typical floor system of a lecture hall is to be designed as a ribbed slab. The joists which are spaced at 400mm are supported by girders. The overall depth of the slab without finishing materials is 300mm. Imposed load of 1.5KN/m$^2$ for partition and fixture is considered in the design. In addition, the floor has a floor finish material of 3cm marble over a 2cm cement screed and it ha 2cm plastering as ceiling. Take the unit weight of ribbed block to be 2KN/m$^2$.

Use:  
C 20/25  
S – 300  
Class 1 works

a) Analyze the ribbed slab system, considering the effects of loading pattern  
b) Design the ribbed slab system
Solution:

Step 1: Material property

Concrete:

\[ f_{ctk, 0.05} = 1.5 \text{MPa} \]

\[ f_{ctm} = 2.2 \text{MPa} \]

\[ \alpha_c = 1.5 \]

\[ f_{ck} = 20 \text{MPa}, f_{cu} = 25 \text{MPa} \]

\[ f_{cd} = \frac{0.85 \times 20}{1.5} = 11.33 \text{MPa} \]

Rebar

\[ f_{yk} = 300 \text{MPa} \]

\[ f_{yd} = \frac{f_{yk}}{1.15} = 260.87 \text{MPa} \]

\[ \varepsilon_{yd} = \frac{f_{yd}}{E_s} = \frac{260.87}{200} = 1.34\% \]

Step 2: Verify if the general requirements for Rib slab are met using Euro Code 2

1. The centers of the ribs should not exceed 1.5 m:
   - This is satisfied, as the center-to-center spacing between the ribs is 400mm.
2. The depth of ribs excluding topping should not exceed four times their average width.
   - Also satisfied as 80 x 4 > 240 mm.
3. The minimum rib width should be determined by consideration of cover, bar spacing and fire resistance
   - BS 8110 code - recommends 125 mm,
   - Assume for this example the conditions are satisfied hence assume requirement satisfied.
4. The thickness of structural topping or flange should not be less than 50mm or one tenth of the clear distance between ribs.
   - 60 mm satisfies this requirement.

Step 3: Loading

Dead load:

- Joist \( \rightarrow 0.2 \times 0.08 \times 25 = 0.4 \)
- Topping \( \rightarrow 0.4 \times 0.06 \times 25 = 0.6 \)
- Floor finish \( \rightarrow 0.4 \times 0.03 \times 27 = 0.32 \)
- Cement Screed \( \rightarrow 0.4 \times 0.02 \times 23 = 0.184 \)
Plastering $\rightarrow 0.4 \times 0.02 \times 23 = 0.184$
Partition and fittings $\rightarrow 0.4 \times 1.5 = 0.6$
Ribbed block $\rightarrow 0.4 \times 2 = 0.8$

$G_k = 3.092 \text{ KN/m}$

Live load:

- $Q_k = 4\text{KN/m}^2 \times 0.4 = 1.6 \text{ KN/m}$

Design load:

- $G_d = 1.35 \times G_k = 1.35 \times 3.092 = 4.174 \text{ KN/m}$
- $Q_d = 1.5 \times Q_k = 1.5 \times 1.6 = 2.4 \text{ KN/m}$

**Step 4: Analysis (for Ribs)**

i) Full design load

![Diagram of full design load](image1)

ii) Maximum support moment [at B and C]

![Diagram of maximum support moment](image2)
iii) For maximum span moment [at span AB and CD]

iv) Maximum span moment [at BC]
v) Only dead load acting

- Moment envelope diagram for the rib

- Maximum reaction envelope

- Minimum reaction envelope
**Step 5. Loading on Girders**

- Assume Width of girders
  
  \[
  \begin{align*}
  A, D & \quad W=300\text{mm} \\
  B, C & \quad W=600\text{mm}
  \end{align*}
  \]

  For all girders \(D=300\text{mm}\)

- Note: the section should be checked for serviceability

  - Self-weight:  
    
    \[
    \begin{align*}
    A & \quad 0.3 \times 0.3 \times 25 = 2.25\text{ KN. m} \\
    B & \quad 0.6 \times 0.3 \times 25 = 4.5\text{ KN. m}
    \end{align*}
    \]

  - Design loads:  
    
    \[
    \begin{align*}
    A & \quad 1.35 \times 2.25 = 3.04\text{ KN. m} \\
    B & \quad 1.35 \times 4.5 = 6.08\text{ KN. m}
    \end{align*}
    \]

**Step 6. Analysis of Girders**

i. For Girder on axis “A” and “D”

- To get to maximum support moment [at 2]

  From the maximum reaction of the Ribs divided by the rib spacing at A and D

  \[
  \frac{10.67}{0.4} = 26.68 + 3.04 = 29.72\text{ KN.m}
  \]

**W=29.72 KN.m**
- To get maximum span moment on girder A & D at 12 & 23
  From the maximum reaction of the Ribs divided by the rib spacing A & D
  \( \frac{10.67}{0.4} = 10.67 \text{ KN.m} \)
  From the minimum reaction of the Ribs divided by the rib spacing \( \frac{6.24}{0.4} = 15.52 \text{ KN.m} \)

- Moment envelop for girder A and D
ii. For Girder on axis “B” and “C”

- Loading: Self-weight = 6.08 KN.m
  Reactions from the ribs (divided by the rib spacing)
  \[
  \frac{29.8}{0.4} = 74.5 \text{ KN.m} \quad \text{and} \quad \frac{18.467}{0.4} = 46.15 \text{ KN.m}
  \]

- To get maximum support moment [at “2”]

- To get maximum span moment [at “12” or “23”]

- 74.5 KN.m
- 46.25 KN.m
- **Moment envelop** diagram for girders on axis B and C

- **Step 7. Loading on the Beam .... Axis 1, 2 and 3**
  - Self-weight width= 200 mm
    Depth= 300 mm
  
  N.B: cross section should be checked for serviceability.
  
  Since there are columns at the intersection of the beams and girders, the beams will only support their own loads.
  
  \[ DL = 0.2 \times 0.3 \times 25 = 1.5 \text{ KN/m} \]
  \[ Gd= 1.35 \times 1.5 = 2.025 \text{ KN/m} \]

- **Beam analysis**
**Step 8: Design**

1. **Rib design**

Cross section at span

\[ h_f = 60 \text{ mm} \]
\[ b_w = 80 \text{ mm} \]
\[ h = 260 \text{ mm} \]

*Take cover 15 mm*

\[ d = 260 - 15 - 6 - \frac{12}{2} = 233 \text{ mm} \]

- **Effective width computation**

\[ b_{eff,i} = 0.2b_i + 0.1l_o \leq 0.2l_o \]

1. **For end span (sagging moment)**

\[ l_o = 0.85l_1 \]
\[ l_o = 0.85 \times 4000 = 3400 \text{ mm} \]
\[ b_1 = b_2 = 160 \text{ mm} \]

\[ b_{eff1} = b_{eff2} = 372 < 680 < b_1 \]
\[ b_{\text{eff}} = \sum b_{\text{eff},i} + b_w \leq b \]

\[ b_{\text{eff}} = 824 \leq 400 \quad \text{NOT OK} \]

\[ b_{\text{eff}} = 400 \text{ mm} \]

II. For interior sagging moment (+ve)

\[ l_o = 0.7l_2 \]
\[ l_o = 0.7 \times 4000 = 2800 \text{mm} \]
\[ b_1 = b_2 = 160 \text{ mm} \]

\[ b_{\text{eff}1} = b_{\text{eff}2} = 312 < 560 < b_1 \]

\[ b_{\text{eff}} = \sum b_{\text{eff},i} + b_w \leq b \]

\[ b_{\text{eff}} = 704 \leq 400 \quad \text{NOT OK} \]

\[ b_{\text{eff}} = 400 \text{ mm} \]

III. For support hogging moment (-ve)

\[ l_o = 0.15(l_1 + l_2) \]
\[ l_o = 1200 \text{mm} \]
\[ b_1 = b_2 = 160 \text{ mm} \]

\[ b_{\text{eff}1} = b_{\text{eff}1} = 152 < 240 < b_1 \]

\[ b_{\text{eff}} = \sum b_{\text{eff},i} + b_w \leq b \]

\[ b_{\text{eff}} = 384 \leq 400 \quad \text{OK} \]

\[ b_{\text{eff}} = 384 \text{ mm} \]

**Note:** However since it is a negative moment the width of the compression zone will be, \( b = 80 \text{mm} \)

- **Design of the T-section**
- **A.** Positive span moment AB and CD
Reinforced concrete structures II – Ribbed slab Example 2

\[ M_{sd} = 9.506 \, KNm \quad b_{eff} = 400 \, mm \quad d = 233mm \quad f_{cd} = 11.33 \, mpa \quad f_{yd} = 260.87 \, mpa \]

\[ \mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{9.506 \times 10^6 \, Nmm}{11.33 \times 400 \times 233^2} = 0.0386 \]

\[ \mu_{sd} < \mu_{sd,lim} = 0.295 \quad \textit{Singly reinforced} \]

\[ K_x = 0.055 \quad X = K_xd = 12.815 \, mm < h_f \quad \textit{design as a rectangular section} \]

\[ K_z = 0.975 \quad Z = K_zd = 227.175mm \]

\[ A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{9.506 \times 10^6 \, Nmm}{260.87 \times 227.15} = 160.40 \, mm^2 \]

\[ A_{smin} = \frac{0.26f_{ctm}}{f_{yk}}b_t d \quad \text{where} \ b_t = b_w \quad d = 233mm \quad f_{ctm} = 2.2 \, mpa \quad f_{yk} = 300 \, mpa \]

\[ A_{smin} = 35.54 \, mm^2 < A_s \quad \textit{OK!} \]

\[ \text{using} \ \Phi 12 \quad a_s = 113.1 \, mm^2 \quad n = \frac{A_s}{a_s} = 1.418 \quad \textit{use 2012bottom bars} \]

B. Negative moment on the rib support B and C

\[ M_{sd} = 11.512 \, KNm \quad b_w = 80 \, mm \quad d = 233mm \quad f_{cd} = 11.33 \, mpa \quad f_{yd} = 260.87 \, mpa \]

\[ \mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{11.512 \times 10^6 \, Nmm}{11.33 \times 80 \times 233^2} = 0.2339 \]

\[ \mu_{sd} < \mu_{sd,lim} = 0.295 \quad \textit{Singly reinforced} \]

\[ K_z = 0.88 \quad Z = K_zd = 205.04 \, mm \]

\[ A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{11.512 \times 10^6 \, Nmm}{260.87 \times 205.04} = 215.22 \, mm^2 \]

\[ A_{smin} = \frac{0.26f_{ctm}}{f_{yk}}b_t d \quad \text{where} \ b_t = b_w \quad d = 233mm \quad f_{ctm} = 2.2 \, mpa \]

\[ f_{yk} = 300 \, mpa \]

\[ A_{smin} = 35.54 \, mm^2 < A_s \quad \textit{OK!} \]

12
using $\varnothing 12$ $a_s = 113.1mm^2$ $n = \frac{A_s}{a_s} = 1.9029$ *use 2Ø12 bars at the top*

C. Span moment between B and C

$M_{sd} = 4.63 \text{ KNm}$ $b_{eff} = 400 \text{ mm} \quad d = 233 \text{ mm} \quad f_{cd} = 11.33 \text{ mpa} \quad f_{yd} = 260.87 \text{ mpa}$

$\mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{4.63 \times 10^6 \text{Nmm}}{11.33 \times 400 \times 233^2} = 0.0188$

$\mu_{sd} < \mu_{sd,lim} = 0.295$ **Singly reinforced**

$K_x = 0.07 \quad X = K_x d = 16.31 \text{ mm} < h_f \text{ design as a rectangular section.}$

$K_z = 0.985 \quad Z = K_z d = 229.505 \text{ mm}$

$A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{4.63 \times 10^6 \text{Nmm}}{260.87 \times 229.505} = 77.33 \text{mm}^2$

$A_{smin} = \frac{0.26f_{ctm}b_t d}{f_{yk}} \quad \text{where} \quad b_t = b_w \quad d = 233 \text{mm} \quad f_{ctm} = 2.2 \text{ mpa} \quad f_{yk} = 300 \text{ mpa}$

$A_{smin} = 35.54 \text{mm}^2 < A_s \text{OK!}$

*using $\varnothing 12$ $a_s = 113.1mm^2$ $n = \frac{A_s}{a_s} = 0.6837$ *use 2Ø12 bottom bars*

**Note:** For the shear design of the ribs, refer to Example 2.3 for ribbed slabs.

2. **Girder design**

a. **Girder at A and D**

- **Positive span moment**

$M_{sd} = 37.728 \text{ KNm} \quad b_w = 300 \text{ mm} \quad D = 300 \text{ mm} \quad f_{cd} = 11.33 \text{ mpa} \quad f_{yd} = 260.87 \text{ mpa}$

$d = 300 - 25 - 8 - \frac{16}{2} = 259 \text{ mm}$

$\mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{37.728 \times 10^6 \text{Nmm}}{11.33 \times 300 \times 259^2} = 0.165$

$\mu_{sd} < \mu_{sd,lim} = 0.295$ **Singly reinforced**

$K_x = 0.907 \quad Z = K_x d = 234.91 \text{ mm}$

$A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{37.728 \times 10^6 \text{Nmm}}{260.87 \times 234.91} = 615.525 \text{mm}^2$
\[ A_{\text{min}} = \frac{0.26f_{\text{ctm}}}{f_{\text{yk}}} b_t d \quad \text{where } b_t = b_w \quad d = 259 \text{ mm} \quad f_{\text{ctm}} = 2.2 \text{ mpa} \quad f_{\text{yk}} = 300 \text{ mpa} \]

\[ A_{\text{min}} = 148.148 \text{mm}^2 < A_s \text{ OK!} \]

using \( \varnothing 16a_s = 200.96 \text{ mm}^2 \quad n = \frac{A_s}{a_s} = 3.0629 \text{ use } 6\varnothing 16 \text{ bottom bars} \]

- **Negative support moment**

\[ M_{sd} = 59.442 \text{ KNm} \quad b_w = 300 \text{ mm} \quad D = 300 \text{ mm} \quad f_{cd} = 11.33 \text{ mpa} \quad f_{yd} = 260.87 \text{ mpa} \]

\[ d = 300 - 25 - 8 - \frac{16}{2} = 259 \text{ mm} \]

\[ \mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{59.442 \times 10^6 \text{Nmm}}{11.33 \times 300 \times 259^2} = 0.260 \]

\[ \mu_{sd} < \mu_{sd,\text{lim}} = 0.295 \text{ Singly reinforced} \]

\[ K_z = 0.841 \quad Z = K_z d = 217.819 \text{ mm} \]

\[ A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{59.442 \times 10^6 \text{Nmm}}{260.87 \times 217.819} = 1046.1 \text{mm}^2 \]

\[ A_{\text{min}} = \frac{0.26f_{\text{ctm}}}{f_{\text{yk}}} b_t d \quad \text{where } b_t = b_w \quad d = 259 \text{ mm} \quad f_{\text{ctm}} = 2.2 \text{ mpa} \quad f_{\text{yk}} = 300 \text{ mpa} \]

\[ A_{\text{min}} = 148.148 \text{mm}^2 < A_s \text{ OK!} \]

using \( \varnothing 16a_s = 200.96 \text{ mm}^2 \quad n = \frac{A_s}{a_s} = 3.0629 \text{ use } 6\varnothing 16 \text{ bottom bars} \]

**b. Girder at B and C**

Positive span moment

\[ M_{sd} = 101.59 \text{ KNm} \quad b_w = 600 \text{ mm} \quad D = 300 \text{ mm} \quad f_{cd} = 11.33 \text{ mpa} \quad f_{yd} = 260.87 \text{ mpa} \]

\[ d = 300 - 25 - 8 - \frac{20}{2} = 257 \text{ mm} \]

\[ \mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{101.59 \times 10^6 \text{Nmm}}{11.33 \times 600 \times 257^2} = 0.226 \]

\[ \mu_{sd} < \mu_{sd,\text{lim}} = 0.295 \text{ Singly reinforced} \]

\[ K_z = 0.867 \quad Z = K_z d = 222.819 \text{ mm} \]

\[ A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{101.59 \times 10^6 \text{Nmm}}{260.87 \times 222.819} = 1747.73 \text{mm}^2 \]
Reinforced concrete structures II – Ribbed slab Example 2

\[ A_{\text{min}} = \frac{0.26 f_{\text{ctm}} b_t d}{f_{\text{yk}}} \quad \text{where } b_t = b_w \quad d = 259 \text{ mm} \quad f_{\text{ctm}} = 2.2 \text{ mpa} \quad f_{\text{yk}} = 300 \text{ mpa} \]

\[ A_{\text{min}} = 148.148 \text{ mm}^2 < A_s \text{ OK!} \]

using \( \phi 20a_s = 314 \text{mm}^2 \quad n = \frac{A_s}{a_s} = 5.566 \) use 6020 bottom bars

Negative support moment

\[ M_{sd} = 161.16 \text{ KNm} \quad b_w = 600 \text{ mm} \quad D = 300 \text{ mm} \quad f_{cd} = 11.33 \text{ mpa} \quad f_{yd} = 260.87 \text{ mpa} \]

\[ d = 300 - 25 - 8 - \frac{20}{2} = 257 \text{ mm} \]

\[ \mu_{sd} = \frac{M_{sd}}{f_{cd} b_d^2} = \frac{161.16 * 10^6 \text{Nm}}{11.33 * 300 * 257^2} = 0.358 \]

\[ \mu_{sd} > \mu_{sd,lim} = 0.295 \quad \text{Doubly reinforced section} \]

\[ K_{z,lim} = 0.814 \]

\[ M_{sd,lim} = \mu_{sd,lim} f_{cd} b_d^2 = 0.295 * 11.33 * 600 * 257^2 = 132.455 \text{ KNm} \]

\[ Z = K_{z,lim} * d = 0.814 * 257 = 209.19 \text{ mm} \]

\[ A_{s1} = \frac{M_{sd,lim}}{Z f_{yd}} + \frac{M_{sd,s} - M_{sd,lim}}{f_{yd} (d - d_2)} = \frac{132.455 * 10^6}{260.87 * 209.19} + \frac{(161.16 - 132.455) * 10^6}{260.87 * (257 - 35)} \]

\[ A_{s1} = 2915.617 \text{mm}^2 \]

\[ \textbf{use 10 } \phi \textbf{20} \]

- Compression reinforcement design

  - Check if the reinforcement has yielded
    \[ \frac{d_2}{d} = \frac{35}{257} = 0.14\varepsilon_{s2} = 2.6\%_0 \quad \text{(read from chart)} \]
    \[ \varepsilon_{s2} = 2.6\%_0 > \varepsilon_{yd} \quad \text{use } f_{yd} = 260.87 \]

  - Calculate the stress in the concrete at the level of compression reinforcement to avoid double counting of area.
    \[ \varepsilon_{c2} = 2.6\%_0 \geq 2\%_0 \quad \text{Therefore, we take} \]
    \[ \varepsilon_c = 3.5\%_0 \quad \text{and } \sigma_{cd,s2} = 11.33 \text{ mpa} \]
Reinforced concrete structures II – Ribbed slab Example 2

\[ A_{s2} = \frac{1}{(\frac{\mu_{sd} - \mu_{sd,lim}}{d - d_s})} \left( \frac{M_{sd} - M_{sd,lim}}{d - d_s} \right) = \frac{1}{(260.87 - 11.33)} \left( \frac{(161.16 - 132.455) \times 10^6}{(257 - 35)} \right) = 518.160 \text{mm}^2 \]

**use 2φ20**

3. **Beams Design**

   i) **Negative support moment**

   \[ M_{sd} = 3.491 \text{KNm} \quad b_w = 200 \text{ mm} \quad D = 300 \text{ mm} \quad f_{cd} = 11.33 \text{ mpa} \quad f_{yd} = 260.87 \text{ mpa} \]

   \[ d = 300 - 25 - 8 - \frac{12}{2} = 261 \text{ mm} \]

   \[ \mu_{sd} = \frac{M_{sd}}{f_{cd} bd^2} = \frac{3.491 \times 10^6 \text{Nmm}}{11.33 \times 200 \times 261^2} = 0.0226 \]

   \[ \mu_{sd} < \mu_{sd,lim} = 0.295 \text{ Singly reinforced} \]

   \[ K_z = 0.985 \quad Z = K_z d = 257.08 \text{ mm} \]

   \[ A_s = \frac{M_{sd}}{f_{yd} Z} = \frac{3.491 \times 10^6 \text{Nmm}}{260.87 \times 257.08} = 52.04 \text{mm}^2 \]

   \[ A_{s,\text{min}} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \quad \text{where} \quad b_t = b_w = 200 \quad d = 261 \text{ mm} \quad f_{ctm} = 2.2 \text{ mpa} \quad f_{yk} = 300 \text{ mpa} \]

   \[ A_{s,\text{min}} = 99.52 \text{mm}^2 > A_s \text{ Not OK!} \]

   \[ \text{using} \ \phi \ 12 a_s = 113.04 \text{mm}^2 \quad n = \frac{A_{s,\text{min}}}{a_s} = 0.88 \text{ use 2φ12 bottom bars} \]

   **Use 2φ12 bottom and top bar for the total length of the beam.**

**Step 8.** **Transverse reinforcement**

Secondary reinforcement is required for temperature and shrinkage.

\[ A_{s2} = 20\% A_{s,\text{min}} \]

\[ A_{s2} = 0.12\% A_{\text{topping}} \]

\[ \text{Spacing} \ S = \frac{ba_s}{A_s} \]

**use φ8 c/c 200 mm**
Step 9. Detailing

*Flexure Reinforcement detailing for Ribs [middle span]*

*Flexure Reinforcement detailing for Ribs [end span]*

*Flexure Reinforcement detailing for Girders [A and D]*
Flexure Reinforcement detailing for Girders [B and C]

4 Ø 18 mm L=
10 Ø 20 mm L=
2 Ø 18 mm L=
6 Ø 20 mm L=

As = 4Ø20 (for construction purpose only)

Girder section A-A

Girder section B-B

b = 600 mm

Flexure Reinforcement detailing for Beam

2Ø12 mm L=
2Ø12 mm L=
2Ø12 mm L=
2Ø12 mm L=
2Ø12 mm L=

As = 2Ø12
As = 2Ø12
All beam sections

Typical Rib section